Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Tuesday 19 May 2020

Morning (Time: 2 hours) | Paper Reference **4MA1/1HR**

Mathematics A

Paper 1HR
Higher Tier

You must have:
Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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1/1/1/

P 6 2 6 5 3 A 0 1 2 4

Pearson

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

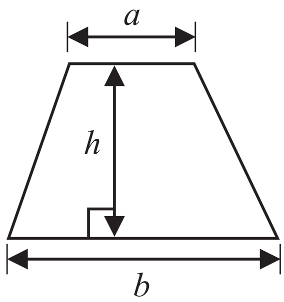
Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

The quadratic equation

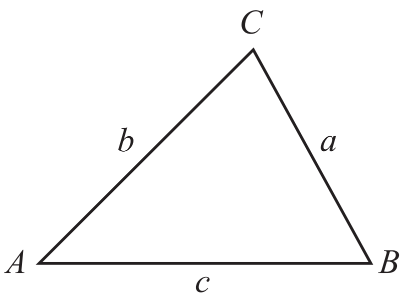
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \frac{1}{2}(a + b)h$



Trigonometry



In any triangle *ABC*

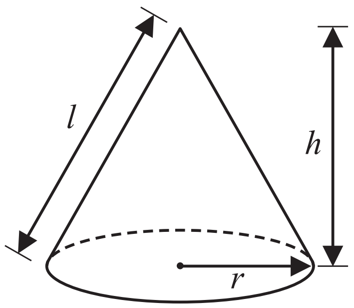
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

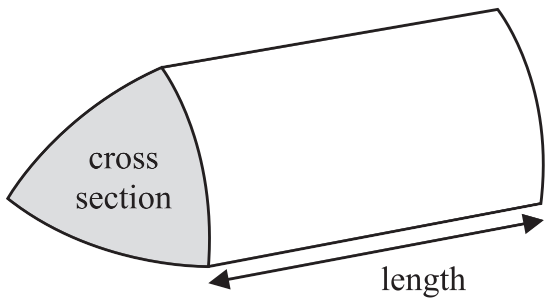
Area of triangle $= \frac{1}{2}ab\sin C$

Volume of cone $= \frac{1}{3}\pi r^2 h$

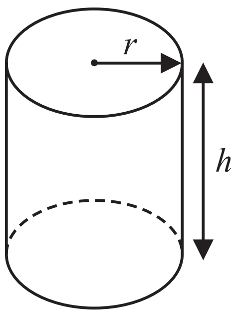
Curved surface area of cone $= \pi r l$



Volume of prism
= area of cross section × length

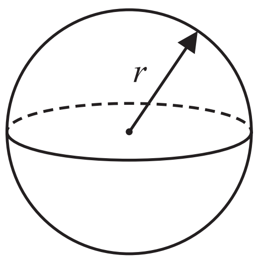


Volume of cylinder $= \pi r^2 h$
Curved surface area
of cylinder $= 2\pi r h$



Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Show that $3\frac{3}{4} \times \frac{7}{9} = 2\frac{11}{12}$

$a\frac{b}{c} = \frac{c \times a + b}{c}$

LHS : $3\frac{3}{4} \times \frac{7}{9}$

$= \frac{15}{4} \times \frac{7}{9}$ (1) (1)

$= \frac{5}{4} \times \frac{7}{3}$

$= \frac{35}{12} = \frac{12}{12} + \frac{12}{12} + \frac{11}{12} = 2\frac{11}{12}$ (RHS)

(1)

(Total for Question 1 is 3 marks)

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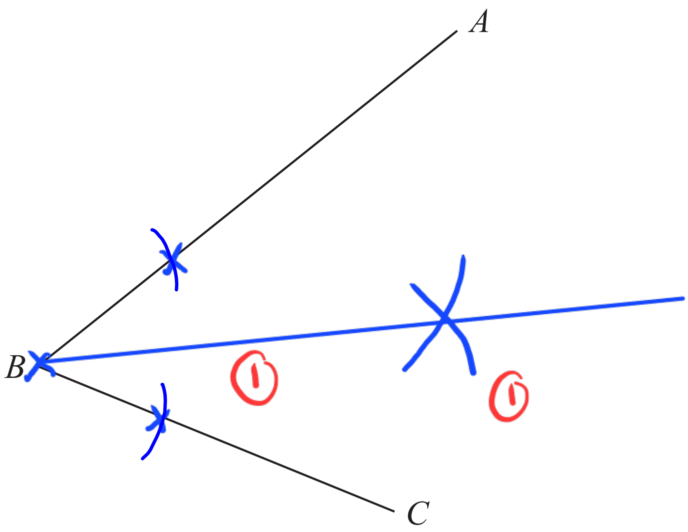
2 Using ruler and compasses only, construct the bisector of angle *ABC*.
You must show all your construction lines.



1. compass on B, mark an arc on AB and AC

2. compass on these marks, draw an arc to the centre from each.

3. connect intersection of arcs to B using a ruler.

(Total for Question 2 is 2 marks)

3 (a) Simplify $h^7 \times h^2$

$h^7 \times h^2 = h^{(7+2)}$

$= h^9$

$a^n \times a^m = a^{n+m}$

$a^n \div a^m = a^{n-m}$

$(a^n)^m = a^{n \times m}$

h^9 1

(1)

$G = c^2 - 4c$

(b) Find the value of G when $c = -5$

substitute $c = -5$ into the equation :

$G = (-5)^2 - 4(-5)$ 1

$= 25 + 20$

$= 45$ 1

$G =$ 45

(2)

(c) Solve $\dfrac{5x-3}{4} = 2x + 3$

Show clear algebraic working.

$\dfrac{5x-3}{4} = 2x+3$ $\times 4$

$5x - 3 = 4(2x+3)$ 1

$5x - 3 = 8x + 12$ $-5x$

$-3 = 3x + 12$

$-15 = 3x$ -12 1

$-5 = x$ $\div 3$

1

$x =$ -5

(3)

(Total for Question 3 is 6 marks)

4 The table gives information about the length of time, in minutes, that each of 60 students took to travel to school on Monday.

Length of time (t minutes)	**Frequency**
$0 < t \leqslant 10$	4
$10 < t \leqslant 20$	10
$20 < t \leqslant 30$	15
$30 < t \leqslant 40$	25
$40 < t \leqslant 50$	6

modal class

(a) Write down the modal class interval.

Modal class = class with highest frequency

$30 < t \leqslant 40$ (1)

(1)

(b) Work out an estimate for the mean length of time taken by these 60 students to travel to school on Monday.
Give your answer correct to one decimal place.

Mean $= \dfrac{\Sigma f x}{\Sigma f}$

interval midpoint × frequency for each class

$$\text{Mean} = \frac{(5 \times 4) + (15 \times 10) + (25 \times 15) + (35 \times 25) + (45 \times 6)}{60} \quad (1)$$

$$= \frac{20 + 150 + 375 + 875 + 270}{60} \quad (1)$$

$$= \frac{1690}{60} \quad (1)$$

$$= 28.2 \ (1 \text{ d.p.}) \quad (1)$$

28.2 minutes

(4)

(Total for Question 4 is 5 marks)

5 In 2017, the population of a village was 7500
In 2019, the population of the village was 8265

(a) Work out the percentage increase in the population of the village from 2017 to 2019

Difference between population in 2017 and 2019 :

$8265 - 7500 = 765$ (1)

Finding percentage increase, $\frac{\text{increment}}{\text{initial value}} \times 100\%$

$\frac{765}{7500} \times 100\% = 10.2\%$ (1) (1)

10.2 .... %
(3)

In a sale, normal prices are reduced by 30%
The sale price of a T-shirt was 31.50 euros.

(b) Work out the normal price of the T-shirt.

Normal price $- \frac{30}{100}$ (normal price) $= 31.50$ (1)

$\frac{70}{100}$ (normal price) $= 31.50$

$\div \frac{70}{100}$

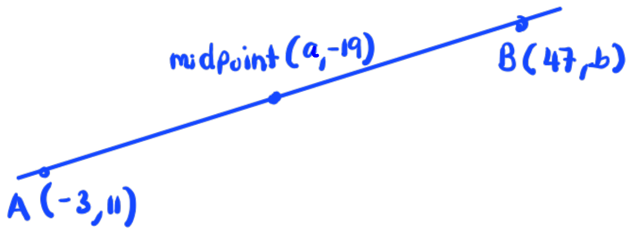
normal price $= 31.50 \times \frac{100}{70}$ (1)

$= 45$ euros (1)

45 .... euros
(3)

(Total for Question 5 is 6 marks)

6 Point A has coordinates (–3, 11)
Point B has coordinates (47, b)
The midpoint of AB has coordinates (a, –19)



Find the value of a and the value of b.

$a = \frac{(-3+47)}{2} = \frac{44}{2} = 22$ 1

$-19 = \frac{(11+b)}{2}$

$-38 = 11 + b$

$b = -38 - 11$

$b = -49$ 1

$a =$ ....22........

$b =$ ....-49........

(Total for Question 6 is 2 marks)

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7 Pedro drove from Toulouse to Montpellier in 2 hours 42 minutes.
He drove at an average speed of 90 km/hour.

Janine drove from Toulouse to Montpellier along the same route as Pedro.
The journey took her 3 hours.

Work out Janine's average speed for the journey.

distance = speed × time

speed = $\frac{\text{distance}}{\text{time}}$

time taken = 2 hours 42 minutes

Speed = 90 km/hour

distance from Toulouse to Montpellier :

$90 \text{ km/h} \times 2 \text{ hour} + \frac{42 \text{ minutes}}{60}$ ← convert minutes to hours

$= 90 \times 2.7$ 1

$= 243 \text{ km}$ 1

Janine's average speed :

$\frac{243 \text{ km}}{3 \text{ hours}} = 81 \text{ km/h}$ 1 1

........81........ km/hour

(Total for Question 7 is 4 marks)

8 Harold bought an antique clock for £1200
The clock increased in value by 8% per year.

Find the value of the clock exactly 3 years after Harold bought the clock.
Give your answer correct to the nearest £.

Initial value : £1200

Year 1 : $£1200 + \frac{8}{100}(1200) = £1296$ 1

Year 2 : $£1296 + \frac{8}{100}(1296) = £1399.68$ 1

Year 3 : $£1399.68 + \frac{8}{100}(1399.68) = £1511.65$

$\approx £1512$ (nearest £) 1

£ 1512

(Total for Question 8 is 3 marks)

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9 A box is put on a horizontal table.

The face of the box in contact with the table is a square of side 1.5 metres.
The pressure on the table due to the box is 34.8 newtons/m^2

Work out the force exerted by the box on the table.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Area : 1.5 m × 1.5 m

= 2.25 m^2 1

Pressure : 34.8 N/m^2

Force = pressure × area

= 34.8 × 2.25 1

= 78.3 N 1

78.3 newtons

(Total for Question 9 is 3 marks)

10 Alex makes 80 cakes to sell.
He makes chocolate cakes, lemon cakes and fruit cakes where

number of chocolate cakes : number of lemon cakes : number of fruit cakes = 3 : 2 : 5

Alex sells
all of the chocolate cakes
$\frac{3}{4}$ of the lemon cakes
$\frac{7}{8}$ of the fruit cakes

The profit he makes on each cake he sells is shown in the table.

Type of cake	Profit per cake he sells
chocolate	£2.00
lemon	£1.70
fruit	£2.40

Work out the total profit that Alex makes from the cakes he sells.

Finding total ratio of cakes :

$3+2+5 = 10$

Finding number of each cakes :

chocolate : $\frac{3}{10} \times 80 = 24$ cakes

lemon : $\frac{2}{10} \times 80 = 16$ cakes (1)

fruit : $\frac{5}{10} \times 80 = 40$ cakes (1)

Finding number of each cakes sold :

chocolate = 24 cakes

lemon = $\frac{3}{4} \times 16 = 12$ cakes (1)

fruit : $\frac{7}{8} \times 40 = 35$ cakes

Total profit made on cakes sales :

$(24 \times 2) + (12 \times 1.70) + (35 \times 2.40)$

$= 48 + 20.4 + 84$ (1)

$= £152.40$ (1)

£ 152.40

(Total for Question 10 is 5 marks)

11 The frequency table gives information about the ages of the 80 people in a train carriage.

Age (*a* years)	**Frequency**
$0 < a \leqslant 20$	9
$20 < a \leqslant 30$	19
$30 < a \leqslant 40$	17
$40 < a \leqslant 50$	18
$50 < a \leqslant 60$	13
$60 < a \leqslant 70$	4

(a) Complete the cumulative frequency table.

Age (*a* years)	**Cumulative frequency**
$0 < a \leqslant 20$	9
$0 < a \leqslant 30$	28
$0 < a \leqslant 40$	45
$0 < a \leqslant 50$	63
$0 < a \leqslant 60$	76
$0 < a \leqslant 70$	80

$9 + 19 = 28$

(1)

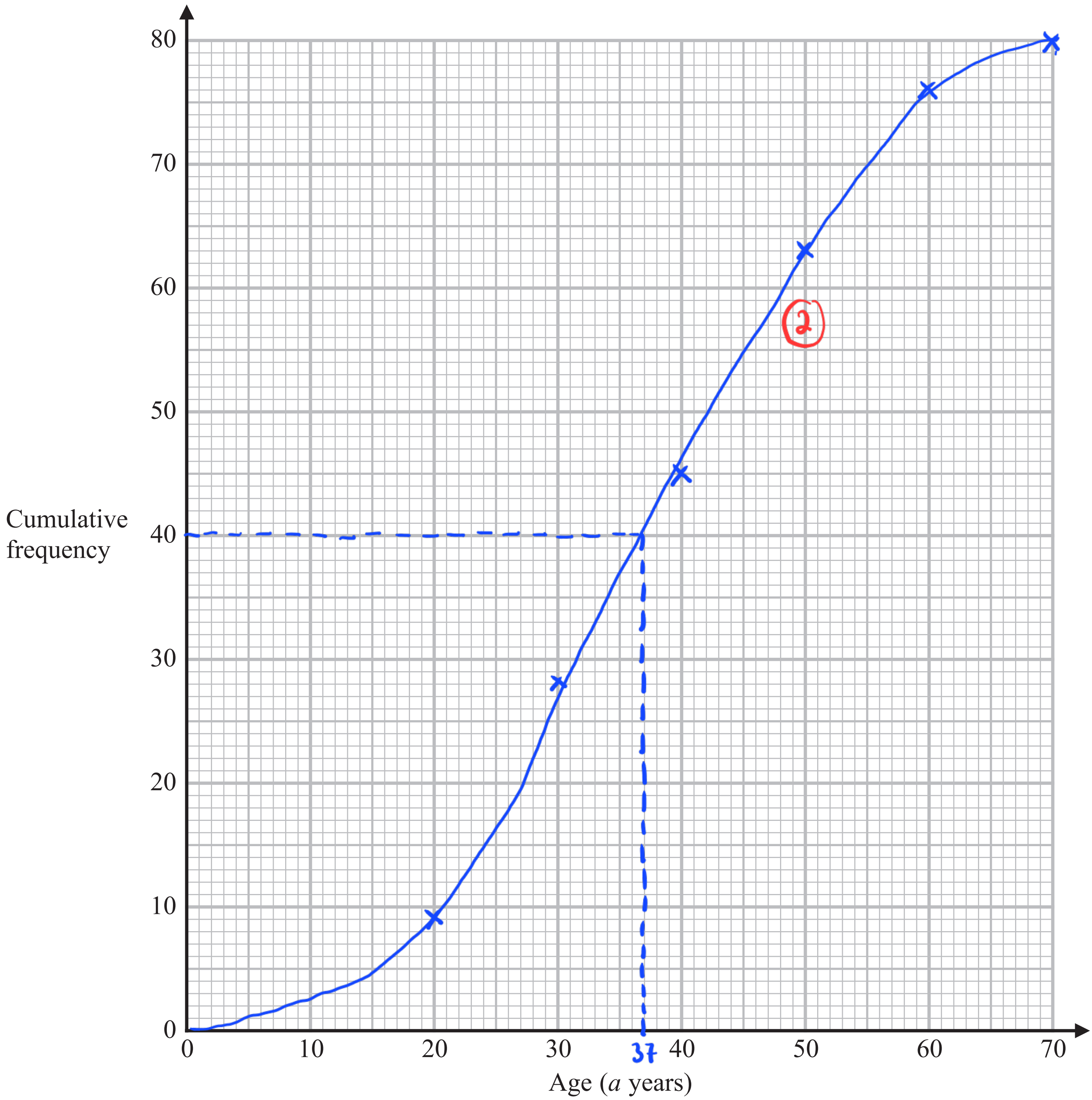
$28 + 17 = 45$

⋮

etc

(1)

(b) On the grid, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median age of the people in the train carriage.

median $= \frac{80}{2} = 40$ (from graph)

37 years

2

(2)

(Total for Question 11 is 5 marks)

12 Solve the simultaneous equations

$$7x + 2y = 5.5$$
$$3x - 5y = 17$$

Show clear algebraic working.

$$7x + 2y = 5.5$$

$-2y$

$$7x = 5.5 - 2y$$

$\div 7$

$$x = \frac{5.5 - 2y}{7} \quad 1 \quad — 1$$

substitute 1 into $3x - 5y = 17$

$$3\left(\frac{5.5 - 2y}{7}\right) - 5y = 17 \quad 1$$

$\times 7$

$$16.5 - 6y - 35y = 119$$

-16.5

$$-6y - 35y = 119 - 16.5$$

$$-41y = 102.5$$

$\div -41$

$$y = -2.5 \quad 1$$

substitute $y = -2.5$ into 1

$$x = \frac{5.5 - 2(-2.5)}{7}$$

$$= 1.5 \quad 1$$

$x =$ 1.5

$y =$ - 2.5

(Total for Question 12 is 4 marks)

13 The curve **C** has equation $y = 5x^3 - x^2 - 6x + 4$

(a) Find $\dfrac{dy}{dx}$

$\dfrac{d}{dx} ax^n = anx^{n-1}$

$\dfrac{dy}{dx} = 15x^2 - 2x - 6$

$\dfrac{dy}{dx} = 15x^2 - 2x - 6$ 2

(2)

There are two points on the curve **C** at which the gradient of the curve is 2

(b) Find the x coordinate of each of these two points.
Show clear algebraic working.

when gradient $= 2$, $\dfrac{dy}{dx} = 2$

$15x^2 - 2x - 6 = 2$ 1 -2

$15x^2 - 2x - 8 = 0$ 1

Finding values of x :

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(15)(-8)}}{2(15)}$ 1

$= \dfrac{2 \pm \sqrt{484}}{30}$

$= \dfrac{2 \pm 22}{30} \Rightarrow \dfrac{24}{30}, \dfrac{-20}{30}$

$= \dfrac{4}{5}, \dfrac{-2}{3}$ 1

$x = \dfrac{4}{5}, -\dfrac{2}{3}$

(4)

(Total for Question 13 is 6 marks)

14 Expand and simplify $(4x + 1)(x - 3)(5x + 6)$

Expanding first 2 brackets :

$(4x+1)(x-3) = 4x^2 - 12x + x - 3$

$= 4x^2 - 11x - 3$ 1

Multiplying first expansion with another bracket :

$(4x^2 - 11x - 3)(5x+6)$

$= 20x^3 + 24x^2 - 55x^2 - 66x - 15x - 18$ 1

$= 20x^3 - 31x^2 - 81x - 18$ 1

$20x^3 - 31x^2 - 81x - 18$

(Total for Question 14 is 3 marks)

15

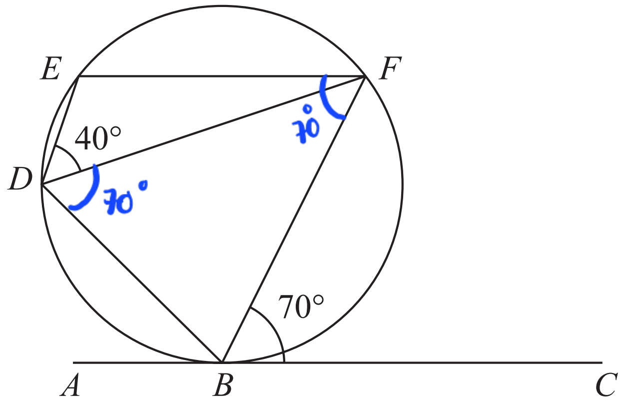


Diagram **NOT** accurately drawn

B, *D*, *E* and *F* are points on a circle.
ABC is the tangent to the circle at *B*.

Angle *EDF* = 40°
Angle *FBC* = 70°

Prove that the tangent *ABC* is parallel to *EF*.
Give a reason for each stage of your working.

$\angle BDF = \angle FBC = 70^\circ$ 1

(alternate segment theorem) 1

$\angle EFB = 180^\circ - \angle EDB$

$= 180^\circ - (40^\circ + 70^\circ)$

$= 180^\circ - 110^\circ$

$= 70^\circ$ 1

(angles opposite to each other in a cyclic quadrilateral sums up to 180°)

∴ since $\angle EFB = 70^\circ$ which is the same as $\angle FBC$,

line EF and line ABC are parallel. 1

($\angle EFB$ and $\angle FBC$ are alternate angles)

(Total for Question 15 is 4 marks)

16 The functions f and g are defined as

$$f : x \mapsto 5x - 7$$

$$g : x \mapsto \frac{5x}{x+4}$$

(a) Write down the value of x that must be excluded from any domain of g

$x = -4 \rightarrow$ since denominator of $g(x)$ cannot be zero

-4 1

(1)

(b) Find gf(2.6)

$f(2.6) = 5(2.6) - 7 = 6$ 1

$gf(2.6) = g(6) = \frac{5(6)}{6+4} = \frac{30}{10} = 3$ 1

3

(2)

(c) Solve $fg(x) = 2$

$fg(x) = 5\left(\frac{5x}{x+4}\right) - 7$ 1

$= \frac{25x}{x+4} - 7$

$fg(x) = 2 = \frac{25x}{x+4} - 7$

$2x + 8 = 25x - 7x - 28$ 1

$\rightarrow 25x - 7x - 2x = 8 + 28$

$16x = 36$

$x = 2.25$ 1

$x =$ 2.25

(3)

(d) Express the inverse function g^{-1} in the form $g^{-1} : x \mapsto \ldots$

$g(x) = \frac{5x}{x+4}$

Let $y = g(x)$. Find x in term of y.

$y = \frac{5x}{x+4}$

$y(x+4) = 5x$ 1

$yx + 4y = 5x$

$4y = 5x - yx$

$4y = x(5-y)$ 1

$x = \frac{4y}{5-y} \Rightarrow g^{-1}(x) = \frac{4x}{5-x}$ 1

$g^{-1} : x \mapsto \frac{4x}{5-x}$

(3)

(Total for Question 16 is 9 marks)

17 The diagram shows a prism *ABCDEFGH* with a horizontal base.

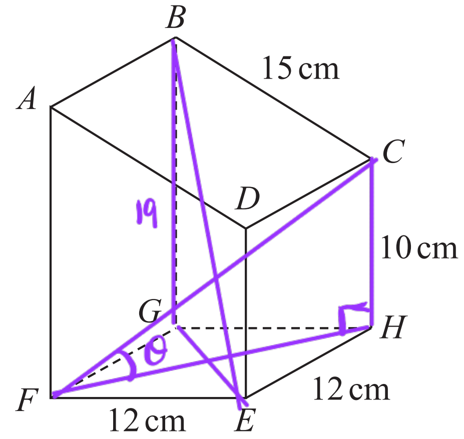


Diagram **NOT** accurately drawn

The base of the prism, *EFGH*, is a square of side 12 cm.
Trapezium *ADEF* is a cross section of the prism where *AF* and *DE* are vertical edges.

$DE = CH = 10$ cm
$AD = BC = 15$ cm

(a) Work out the size of the angle between *CF* and the base *EFGH*.
Give your answer correct to one decimal place.

diagonal FH $= \sqrt{12^2+12^2}$

$= 12\sqrt{2}$ cm (1)

By using trigonometry :

$\tan\theta = \dfrac{10}{12\sqrt{2}}$ (1)

$\theta = 30.5^\circ$ (1)

30.5 °

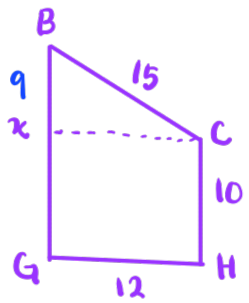
(3)

(b) Work out the length of *BE*.
Give your answer correct to one decimal place.

$Bx = 15^2 - 12^2$

$= 9$ cm

$BG = 9 + 10 = 19$ cm (1)



$BE = \sqrt{BG^2 + GE^2}$

$= \sqrt{19^2 + (12\sqrt{2})^2}$ (1)

$= \sqrt{649}$

$= 25.5$ cm (1)

BGE forms a right angle triangle.
We can use Pythagoras Theorem
to solve for BE.

25.5 cm

(3)

(Total for Question 17 is 6 marks)

18 In an arithmetic series, the 6th term is 39
In the same arithmetic series, the 19th term is 7.8

Work out the sum of the first 25 terms of the arithmetic series.

$T_n = a + (n-1)d$

$T_6 = 39 = a + 5d$ — (1)

$T_{19} = 7.8 = a + 18d$ — (2)

Substitute (1) into (2) :

$7.8 = (39 - 5d) + 18d$

$7.8 - 39 = 13d$

$-31.2 = 13d$ (1)

$d = \frac{-31.2}{13} = -2.4$

Substitute $d = -2.4$ into (1)

$39 = a + 5(-2.4)$

$39 = a - 12$

$a = 39 + 12$

$= 51$ (1)

Sum of first 25 term : $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{25} = \frac{25}{2}\left[2(51) + (24)(-2.4)\right]$ (1)

$= 12.5(102 - 57.6)$

$= 555$ (1)

555

(Total for Question 18 is 4 marks)

19 The diagram shows rectangle $ABCD$ with rectangle $EFGH$ cut out to form the shaded region.

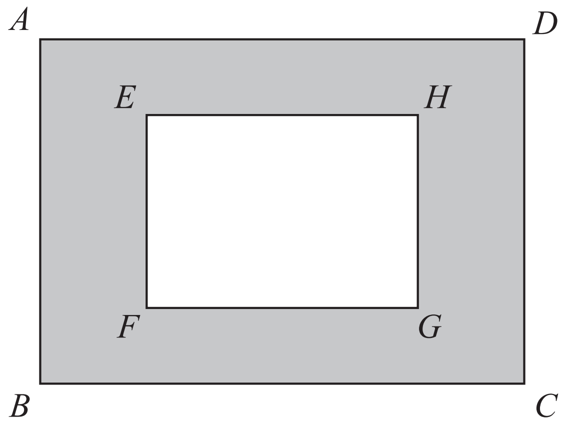


Diagram **NOT** accurately drawn

$AD = 8.3$ cm correct to one decimal place
$DC = 7.2$ cm correct to one decimal place
$EH = 6.2$ cm correct to one decimal place
$HG = 5.3$ cm correct to one decimal place

Work out the upper bound of the area of the shaded region.
Show your working clearly.

To get upper bound of the area of shaded region :

1 we get the upper bound of larger rectangle (area)

2 we get the lower bound of smaller rectangle (area)

3 1 minus 2 to get the area of shaded region

1 Area of ABCD : 8.35 cm $\times$ 7.25 cm

$= 60.5375$ cm^2 1

2 Area of EFGH : 6.15 cm $\times$ 5.25 cm

$= 32.2875$ cm^2

3 Area of shaded region : $60.5375 - 32.2875$ 1

$= 28.25$ cm^2 1

28.25 .... cm^2

(Total for Question 19 is 3 marks)

20 A curve has equation $y = f(x)$
There is only one maximum point on the curve.
The coordinates of this maximum point are $(-3, 4)$

Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x) - 6$

- value of x is not changing
- value of y translated down by 6 unit.

$(-3, 4-6) = (-3, -2)$

(-3 , -2) 1

(ii) $y = f(2x)$

- y-coordinate remains the same
- x-coordinate is divided by 2

(-1.5 , 4) 1

(Total for Question 20 is 2 marks)

21 Given that $M = \dfrac{18^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{12^2}$

find the values of n for which $M = 2$

$$M = \frac{18^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{12^2}$$

(1)

$$2 = \frac{(2 \times 3^2)^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{2^4 \times 3^2} \quad (1)$$

$$2^1 = \frac{2^{4n} \times 2^{3n^2-18n} \times 3^{8n} \times 3^{2-8n}}{2^4 \times 3^2}$$

$$2^1 = 2^{3n^2-14n-4} \times 3^{8n-8n+2-2}$$

$$2^1 = 2^{3n^2-14n-4} \times 1$$

$$1 = 3n^2 - 14n - 4$$

$$3n^2 - 14n - 5 = 0 \quad (1)$$

$$n = \frac{14 \pm \sqrt{(-14)^2 - 4(3)(-5)}}{2(3)} \quad (1)$$

$$= \frac{14 \pm \sqrt{256}}{6} \Rightarrow \frac{14 \pm 16}{6}$$

$$n = \frac{30}{6}, \quad n = -\frac{2}{6}$$

$$= 5, \quad -\tfrac{1}{3} \quad (1)$$

$-\tfrac{1}{3}, 5$

(Total for Question 21 is 5 marks)

22 The diagram shows a regular octagon *ABCDEFGH*.

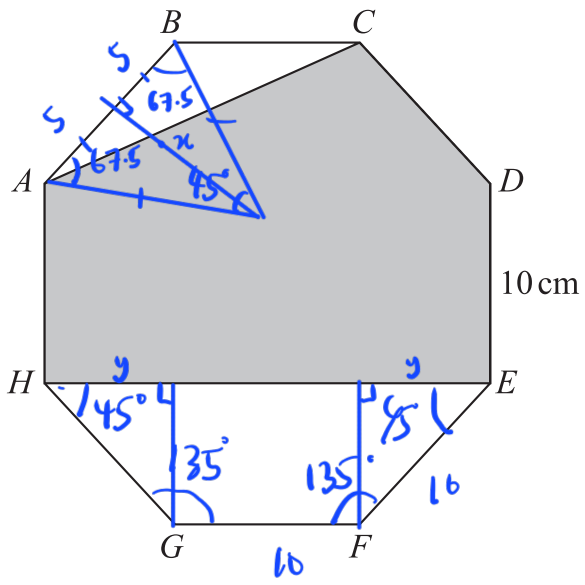


Diagram **NOT** accurately drawn

Each side of the octagon has length 10 cm.

Find the area of the shaded region *ACDEH*.
Give your answer correct to the nearest cm^2

interior angle of octagon : $\frac{(8-2)}{8} \times 180° = 135°$ (1)

split octagon into 8 triangles

Find x : $x = 5 \tan 67.5° = 12.07106\ldots$

Area of triangle $= \frac{1}{2} \times 10 \times 12.07106\ldots = 60.355\ldots$

Total area of octagon = 8 × 1 area of triangle

Area of octagon $= 8 \times 60.355\ldots$
$= 482.84\ldots$ (1)

Area of triangle ABC : $\frac{1}{2} \times 10 \times 10 \times \sin 135° = 25\sqrt{2} = 35.355\ldots$ (1)

Find y : $y = 10 \cos 45° = 5\sqrt{2}$

Length of HE : $2 \times 5\sqrt{2} + 10 = 10\sqrt{2} + 10$ (1)

Area of trapezium : $\frac{1}{2} \times (10\sqrt{2} + 10 + 10) \times 10 \sin 45°$

$= 120.71\ldots$ (1)

Area of shaded region = Area of octagon − area of triangle ABC − Area of trapezium

Area of shaded region = $482.84... - 35.355... - 120.71...$

$= 326.77...$

$= 327 \text{ cm}^2$ (nearest cm^2)

1

327 cm^2

(Total for Question 22 is 6 marks)

Turn over for Question 23

23 In a bag, there are only

3 blue beads
4 white beads
and x orange beads.

Jean is going to take at random two beads from the bag.

The probability that Jean will take two beads of the same colour is $\frac{3}{8}$

Find the total number of beads in the bag.
Show clear algebraic working.

Probability of taking two beads of same colour : (let n = total number of beads)

Blue : $\frac{3}{n} \times \frac{2}{n-1} = \frac{6}{n(n-1)}$

White : $\frac{4}{n} \times \frac{3}{n-1} = \frac{12}{n(n-1)}$

Orange : $\frac{n-7}{n} \times \frac{n-8}{n-1} = \frac{(n-7)(n-8)}{n(n-1)}$

Combine : $\frac{6}{n(n-1)} + \frac{12}{n(n-1)} + \frac{(n-7)(n-8)}{n(n-1)} = \frac{3}{8}$ 1

$6 + 12 + n^2 - 15n + 56 = \frac{3}{8}(n^2 - n)$

$48 + 96 + 8n^2 - 120n + 448 = 3n^2 - 3n$ 1

$5n^2 - 117n + 592 = 0$ 1

$n = \frac{117 \pm \sqrt{(-117)^2 - 4(5)(592)}}{2(5)}$

$= \frac{117 \pm 43}{10}$

$= 16, 17.4$

n = 16 since no. of beads should be whole number 1

16

(Total for Question 23 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS